

DERIVING THE VELOCITY DISTRIBUTION OF METEORIODS  
FROM THE MEASURED METEOROID IMPACT DIRECTIONALITY  
ON THE VARIOUS LDEF SURFACES

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SUMMARY

Because of spacecraft orbital motion about the Earth, a much higher flux of meteoroids is expected to strike spacecraft surfaces that face in the direction of spacecraft motion (apex direction) than would strike antapex-facing, or trailing edge, surfaces. Impact velocities are also higher on apex-facing surfaces compared to antapex-facing surfaces which further increases the apex/antapex ratio of spatial density of impact craters of a given size. Measurements of the areal densities of impact craters on the different LDEF surfaces should give important clues about the velocity distribution, and therefore the origins, of meteoroids. Preliminary results so far reported from LDEF investigations appear to best support the meteoroid velocity distributions derived by Erickson and by Kessler, which would lead to a mean impact velocity on the LDEF spacecraft of about 19 km/s.

INTRODUCTION

It is likely that meteoroids do not enter the Earth's atmosphere with equal probability from all directions. The true directional distribution, however, is not yet clear. Do more meteoroids, for example, approach the Earth from its direction of motion around the Sun (also called the "heliocentric apex" direction, or the "morning" side of the Earth), than from other directions? Southworth and Sekanina (ref. 1), after correcting their radar observations of meteoroids entering the terrestrial atmosphere for various experimental biases, obtain a flux—at constant meteoroid mass—with a peak in the heliocentric antapex direction (the "evening" side). There were also "peaks" in other directions, but not in the heliocentric apex direction. There remains some uncertainty, however, as to whether or not they have correctly accounted for all experimental biases. The true directional distribution of approach may also depend on meteoroid mass.

We note, however, that any given surface on the LDEF spacecraft will, over time, face in a large variety of directions relative to, say, the Earth-Sun line. This is a result of: (1) Normal vectors to the apex (leading), antapex, and space-facing surfaces of LDEF sweep through 360 degrees during each orbit about the Earth; (2) the ascending node of the LDEF orbit plane precesses with respect to the Earth-Sun line by nearly 8° per day; and (3) the spin axis of the Earth is inclined 23.5 degrees to the Earth's orbital axis about the Sun (see Fig. 1). This means that meteoroids arriving from a single

heliocentric longitude and latitude throughout the year will, before LDEF motion is taken into account, impact from a great variety of directions relative to the spacecraft geocentric apex direction.

This fact suggests the following assumption: "before satellite motion is taken into account, meteoroid radiants of every entry velocity will appear to arrive in uniform numbers from every direction not shielded by the Earth" (see also ref. 2). This will be called the "randomness" assumption for the distribution of meteoroid arrival directions. The assumption would be rigorously true, of course, if meteoroids actually enter the terrestrial atmosphere uniformly from all directions. When the actual rather broad, but poorly known, distribution of atmospherically-observed meteor radiants is considered, the assumption may be approximately true. The actual distribution of impact velocities and radiants on LDEF (or any orbiting satellite) is then obtained by permitting the LDEF spacecraft to move through this assumed random distribution of radiants with its Earth orbital velocity (similar to motion through a very rarified isotropic gas). This gives rise to a new "apparent" distribution of impact radiants and velocities relative to the spacecraft apex direction. The randomness assumption is one that makes it possible to deduce relative cratering rates on various LDEF surfaces as a function of the meteoroid velocity distribution. This, in turn, makes it possible to either test the assumption or to find out which meteoroid velocity distribution is best by comparison with the observed data. As more is learned about the true meteoroid directionality with respect to the Earth, the "randomness" assumption can be changed to fit the new facts.

## ANALYSIS AND RESULTS

Consider an infinitesimal flux,  $dF_{va}$ , of meteoroids approaching the LDEF spacecraft location from a small solid angle  $\sin\theta d\theta d\phi$  and in a small velocity interval  $dv$ , where  $\theta$  is the angle of approach with respect to the spacecraft apex direction and  $\phi$  is the azimuth angle around the apex direction, with  $\phi = 0$  when pointed radially away from the Earth;  $v$  and  $\theta$  are taken to be the velocity and apex angle before spacecraft motion is taken into account. The subscript "a" refers to the angular dependence of  $dF$ . Then, by the "randomness" assumption of the previous paragraph,

$$dF_{va} = [1/(4\pi - \Omega_E)] \sin\theta d\theta d\phi n(v) dv, \quad (1)$$

where  $\Omega_E$  is the solid angle subtended by the Earth and the denser part of its atmosphere, and  $n(v)$  is the distribution of velocities with which meteoroids are observed to enter the top of the atmosphere. For an effective altitude of LDEF of 460 km, and an effective height of the atmosphere of 150 km (below which it is assumed that meteoroids cannot first pass and then strike LDEF), the top of the atmosphere appears 17.3 degrees below the local horizontal. Then  $\Omega_E = 4.41$  steradians. That is, the Earth plus its atmosphere shields out 35.1% of the sky from meteoroid entry.  $n(v)$  is normalized so that

$$\int_v n(v) dv = 1. \quad (2)$$

When  $dF_{va}$  is integrated over all angles  $\theta$  and  $\phi$  (in radians) not shielded by the Earth, and over all velocities  $v$ , one obtains unity, which means  $F_{va}$  is also normalized. When  $\theta$  is larger than  $\theta_m$ , where  $\theta_m = 17.3$  degrees, then the limits of integration of  $\phi$  are from  $-\phi_m$  to  $+\phi_m$ , where

$$\phi_m = \pi/2 + \arctan[\sin\theta_m/(\cos^2\theta_m - \cos^2\theta)^{0.5}], \quad (3)$$

which gives the range of  $\phi$  angles for which the Earth is not in the field of view. When  $\theta$  is less than  $\theta_m$ ,  $\phi$  ranges over  $2\pi$  radians.

Now consider the spacecraft in motion with its regular Earth orbital velocity,  $v_s$  ( $v_s = 7.68$  km/s at 460 km altitude). The velocity,  $v_r$ , with which the meteoroid and spacecraft approach each other is given by  $v_r = v - v_s$ , where  $v_r, v$ , and  $v_s$  are vector velocities, and  $v$  is the meteoroid velocity. The apparent angle  $\psi$ , relative to the spacecraft apex direction, with which the meteoroids will appear to impact the moving spacecraft is obtained from

$$\cos\psi = (v\cos\theta + v_s)/v_r, \quad (4)$$

where

$$v_r = (v_s^2 + v^2 + 2v_s v \cos\theta)^{0.5}. \quad (5)$$

If  $dF_{va}$  is divided by  $v$ , we obtain the spatial density  $dN(v, \theta, \phi)$  of meteoroids arriving from directions  $\theta$  to  $\theta + d\theta$ ,  $\phi$  to  $\phi + d\phi$ , and in velocity interval  $v$  to  $v + dv$ . That is

$$dN(v, \theta, \phi) = dF_{va}/v = N(v, \theta, \phi) \sin\theta d\theta d\phi dv, \quad (6)$$

and, using Equation (1),

$$N(v, \theta, \phi) = [1/(4\pi - \Omega_E)] n(v)/v \quad (7)$$

for all directions not shielded by the Earth. From directions shielded by the Earth,  $N(v, \theta, \phi) = 0$ . Our "randomness" assumption means that  $N$  has no  $\theta$  or  $\phi$  dependence, except for Earth shielding.

When the spatial density of a differential velocity-angle subgroup of particles is multiplied by the velocity  $v_r$  relative to a spacecraft, we obtain the differential flux (number/(area - time)) of meteoroids impacting on the spacecraft at velocity  $v_r$  to  $v_r + dv_r$  and from directions  $\psi$  to  $\psi + d\psi$  and  $\phi$  to  $\phi + d\phi$ .  $\psi$  and  $v_r$  are obtained from Equations (4) and (5), respectively. In equation form

$$dF_r(v_r, \psi, \phi) = F_r(v_r, \psi, \phi) \sin\psi d\psi d\phi dv_r = dF_{va} v_r/v, \quad (8)$$

where  $F_r$  is the flux per unit solid angle and per unit velocity that impacts the orbiting spacecraft.

This equation can be solved for  $dF_r$ , and hence for  $F_r$ , if the velocity distribution  $n(v)$  in Equation (7) is known. Dohnanyi (ref. 3), Erickson (ref. 4), Kessler (ref. 5), and Southworth and Sekanina (ref. 1) independently analyzed different observed distributions of atmospheric meteor entry velocities, corrected them for various selection effects, and presented meteoroid velocity distributions at constant meteoroid mass. Zook (ref. 6) assembled these different distributions together in a single paper and made approximate fits of analytical formulas to the Erickson and to the Southworth and Sekanina results (Dohnanyi had already represented his results analytically). The resulting velocity distributions are shown graphically in Fig. 2. The Kessler distribution is so similar to the Erickson distribution, that I will call the mathematical fit to the Erickson distribution the "Erickson-Kessler" distribution. References 3, 4, and 5 studied different sets of photographic meteor observations, and reference 1 studied radar meteors. It is assumed that the differences between these derived velocity distributions is due to different techniques in correcting for sensor biases, in using different data sets, and in possible true differences between photographic meteors and the smaller mass radar meteors. These different published velocity distributions give us some feel for the uncertainty in determining a "true" velocity distribution at constant meteoroid mass.

In this paper I use three separate velocity distributions for  $n(v)$  in Equation (7), to see if predicted crater statistics around LDEF depend much on the  $n(v)$  used. They are the Dohnanyi, the Erickson-Kessler, and the Southworth & Sekanina distributions (formulas given in ref. 6). Equation (8) is numerically solved by uniformly incrementing all  $v$ ,  $\theta$ , and  $\phi$  values, weighting each  $(\theta, \phi)$  angle by  $\sin\theta$ , and each velocity by  $n(v)$  and by  $v_r/v$ ; and by the differentials  $d\theta, d\phi, dv$ , and then storing the resulting numeric sums of the  $dF_r$  in small "bins", or intervals of  $(\psi, \phi, v_r)$ .  $F_r(v_r, \psi, \phi)$  is then found by dividing the summed  $dF_r$  in a given interval by  $\sin\psi d\psi d\phi$ , the differential solid angle interval from which meteoroids "appear" arrive at a spacecraft orbiting with velocity  $v_s$ . The input  $n(v)$  have been very modestly modified from ref. 6, by accounting for gravity-induced increases in meteoroid velocities from LDEF altitude of 460 km to the top of the atmosphere at 100 km where meteor measurements were made. The  $n(v)$  were then renormalized. It is found that, when one integrates over all angles and velocities in Equation (8), the result does not equal 1 (i.e.,  $F_r$  is not normalized). Instead, the number ranges from 1.06 for the Dohnanyi distribution to 1.10 for the Southworth and Sekanina distribution. The reason for this is that a unit flux of meteoroids (at constant mass) on a spherical spacecraft at rest with respect to the Earth is increased by several percent on a spacecraft moving with orbital velocity. The increase, as would be expected, is greater for low velocity meteoroids than for high velocity meteoroids.

If one sums only over all angles, and again normalizes, one obtains the velocity distribution with which meteoroids strike a spherical (or randomly tumbling) orbiting spacecraft. These are shown in Fig. 3 for each of the velocity distributions. It is noted that mean impact velocities have increased by about 2 km/s in each case. It is interesting to note that the percentage increase in mean relative velocity, in going from a stationary spacecraft to one with the orbital velocity, is greater than the percentage increase in impacting flux.

If, in Equation (8), one integrates  $\psi$  only over 0 to 90 degrees, and sums over all allowable  $v_r$  and  $\phi$ , one obtains the meteoroid flux, at constant meteoroid mass, striking a flat plate with its normal facing in the forward direction. By similarly integrating  $\psi$  over 90 to 180 degrees, one obtains the

corresponding flux striking a flat plate facing in the antapex direction. The resulting ratios of fluxes on flat plates—apex to antapex—for the different velocity distributions are as follows: 5.7 for Dohnanyi, 7.2 for Erickson-Kessler, and 9.2 for Southworth and Sekanina. Not only are the fluxes different on apex and antapex-facing plates, so are the impact velocities. In Fig. 4 velocity distributions are shown separately (after normalizing) on apex and antapex-facing plates, where the Erickson-Kessler velocity distribution was the input distribution used. These distributions are valid for constant meteoroid mass and not for a constant resulting crater diameter.

Also of interest is the angular distribution with which meteoroids are expected to strike an orbiting spacecraft. To find this distribution, Equation (8) is summed over all meteoroid velocities. One then obtains the angular distributions (not normalized) shown in Fig. 5 for the Dohnanyi and Southworth and Sekanina distributions. These distributions are per unit solid angle and are valid at a given meteoroid mass and for directions not shielded by the Earth. The Erickson-Kessler distribution would lie between the other two.

Finally, however, one needs to know how the spatial density of impact craters around LDEF depends on the assumed velocity distribution of meteoroids--as crater frequency versus crater diameter and versus location on LDEF are the observed quantities. Presumably, the velocity distribution that gives rise to results that best fits the observed data is the "correct" one (and assuming the 'randomness' assumption is nearly correct). To carry out this task we use the penetration equation for 6061-T6 aluminum from ref. 7, which is as follows:

$$P = 0.42m^{0.352}\rho^{1/6}v^{2/3}, \quad (9)$$

where  $P$  is the penetration depth in cm,  $m$  is the meteoroid mass in g,  $\rho$  is the meteoroid mass density in  $\text{g/cm}^3$ , and  $v$  is the normal impact velocity in km/s. For this study, I assume  $\rho = 2 \text{ g/cm}^3$ , and rewrite the equation to give

$$P = 0.48d^{1.056}(v\cos\theta)^{2/3}, \quad (10)$$

where  $d$  is the meteoroid diameter in cm and  $\theta$  is angle with respect to the normal with which meteoroids impact a surface. For a moving spacecraft,  $v$  should be replaced by  $v_r$ . Crater diameter  $D$  is assumed to be twice the penetration depth  $P$ . For a normal impact ( $\theta = 0$ ) at  $v = 20 \text{ km/s}$ , the meteoroid masses required to generate 100 and 500  $\mu\text{m}$  in diameter craters are, respectively,  $8.5 \times 10^{-9} \text{ g}$  and  $8.2 \times 10^{-7} \text{ g}$ . From ref. 8, the slopes of the  $\log(\text{flux})$  versus  $\log(\text{mass})$  curve at these meteoroid masses are -0.48 and -0.90, respectively.

Because meteoroids strike from the apex direction at typically higher velocities (due to spacecraft orbital motion) than from the antapex direction, smaller—and more numerous—meteoroids make more impact craters on the apex-facing surface than on the antapex-facing surface. This means that the ratio of the number of impact craters of a fixed diameter on the apex side compared to number of the same diameter on the antapex side depends not only on relative fluxes at constant mass, but on the slope of the  $\log(\text{meteoroid flux})$  versus  $\log(\text{meteoroid mass})$  curve. The analysis presented here depends on this effect and follows the technique used by Naumann (ref. 9) in accounting for the increased meteoroid flux at small meteoroid masses.

The Naumann analysis also applies to meteoroids striking surfaces at oblique angles. To make a crater of a certain fixed depth, or diameter, larger—and less numerous—meteoroids are required at oblique angles on a surface than at perpendicular, or normal, impact (at fixed velocity). The oblique angle effect should show up quite dramatically in the relative crater frequency of a given size crater on the "Top," or space-facing end of LDEF, compared to the "Bottom," or Earth-facing end of LDEF. There is a lesser effect from Top to "Side" (North or South-facing). The spacecraft orbital velocity should have no effect on these particular ratios, unless there is local shielding by the spacecraft. This is because the normal component of impact velocity has not been changed (although impacts will usually be at more oblique angles). Impacts also tend to occur at more normal incidence on the apex-facing surface than on the antapex-facing surface which, again, adds to enhance the "cratering" flux in the apex direction.

Table 1. Relative meteoroid crater production rates on LDEF as a function of crater diameter (on 6061-T6 A1), and as a function of the velocity distribution used. Meteoroid mass and the slope of the log (meteoroid flux) versus log (meteoroid mass) curve are also given at each crater diameter.

Crater dia. ( $\mu\text{m}$ )	Mass (g)	Slope	Vel. Dist.	Apex	Top	Side	Antapex	Bottom
500	$7.8 \times 10^{-7}$	-0.90	Dohnanyi	12.2	6.4	4.7	1	0.06
100	$8.1 \times 10^{-9}$	-0.48	Dohnanyi	9.9	5.9	4.2	1	0.13
500	$7.8 \times 10^{-7}$	-0.90	E-K	19.2	8.7	6.4	1	0.08
100	$8.1 \times 10^{-9}$	-0.48	E-K	14.4	7.6	5.4	1	0.17
500	$7.8 \times 10^{-7}$	-0.90	S&S	32.8	12.8	9.4	1	0.12
100	$8.1 \times 10^{-9}$	-0.48	S&S	21.2	10.1	7.2	1	0.23

Table 1 depicts the relative number/area of craters expected, depending on the crater diameter and meteoroid velocity distribution used, on each of six different surfaces facing in perpendicular directions (including north and south-facing surfaces) of LDEF. The number/area on the antapex-facing surface is taken to be 1, so all other surfaces show meteoroid fluxes relative to the antapex direction. Spacecraft motion and oblique impacts are accounted for, and the angle and velocity dependencies of (9) are integrated over all angles and velocities. The three velocity distributions used are those of Dohnanyi (3), Erickson (4)-Kessler (5) (=E-K), and Southworth and Sekanina (7) (=S&S).

As previously mentioned, it was assumed that LDEF is at a mean altitude of 460 km above the Earth, and that the effective atmospheric height is 150 km, below which meteoroids can not pass before impacting LDEF. This means that the minimum angle to the normal with which meteoroids can impact the Bottom side of LDEF is 72.5 degrees, before spacecraft velocity is considered. The reason for the strikingly high ratio (about 105) for the frequency of 500  $\mu\text{m}$  wide craters on the Top surface of LDEF compared to the Bottom of LDEF is due to the steep slope of the flux-mass curve at these large meteoroid masses. It was assumed, in all cases, that there was no local spacecraft shielding.

## DISCUSSION

Jackson and Zook (10) find that dust particles from the main belt of asteroids are expected to have mean velocities of 6 to 7 km/s relative to the Earth by the time they have drifted to Earth encounter (before the Earth's gravitational acceleration is accounted for). These average velocities would suggest that dust from the asteroid belt comprises from 5% (Dohnanyi vel. dist.) to 30% (S&S vel. dist.) of the meteoritic dust at 1 AU, before considering the gravitational enhancement of the flux by the Earth (11). Singer et al. (12) have sensed beta meteoroids on the antapex surface of LDEF. If the flux of beta's can also be measured on other surfaces, it should be possible to derive an "effective" velocity for these meteoroids; this would be an important experimental determination. The directionality of beta meteoroids may also be determined.

It will be of great interest to determine which one of the meteoroid crater distributions given in Table 2 above best fits the actual meteoroid impact crater data on LDEF (after orbital debris impacts have been accounted for). Or, do any of them fit? Beta meteoroids, for example, may travel at much higher velocities, on average, than other meteoroids. They also may not satisfy the "randomness" assumption very well, as they may mostly arrive at relatively small angles to the ecliptic. I note, finally, that one may make some other assumption than the randomness assumption, and again carry through the analyses that have been carried out in this paper. LDEF may help us, in this regard.

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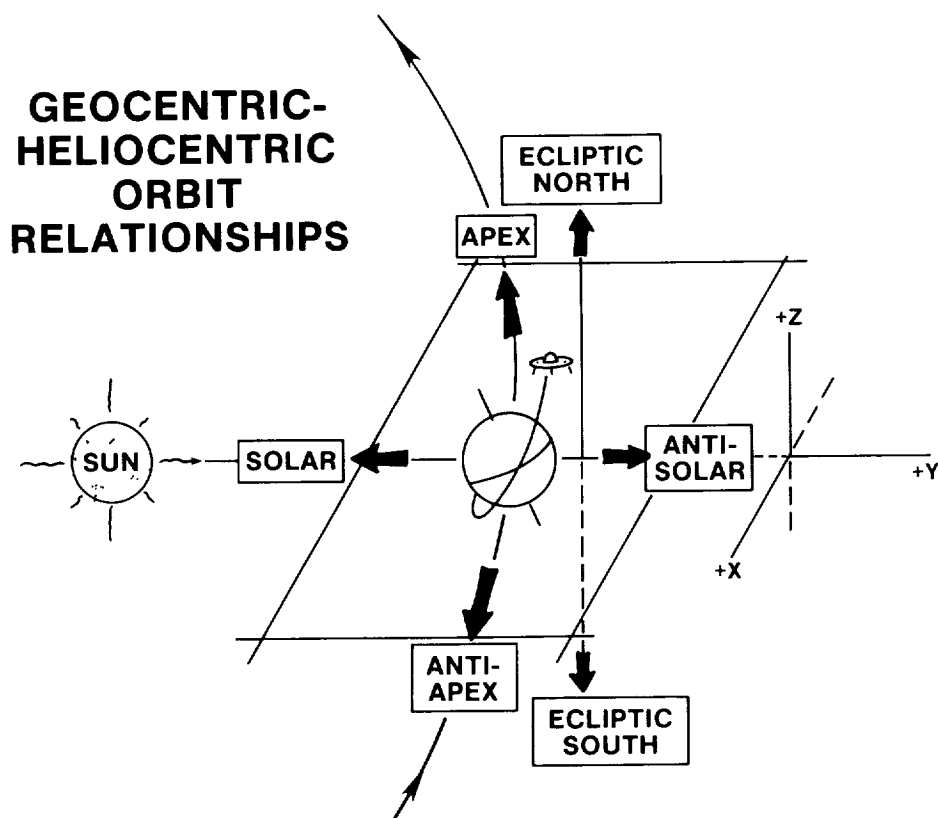


Figure 1. Spacecraft geocentric orbit and its relationship to heliocentric space.

## VELOCITY DISTRIBUTION OF METEOROIDS (NORMALIZED) AT EARTH

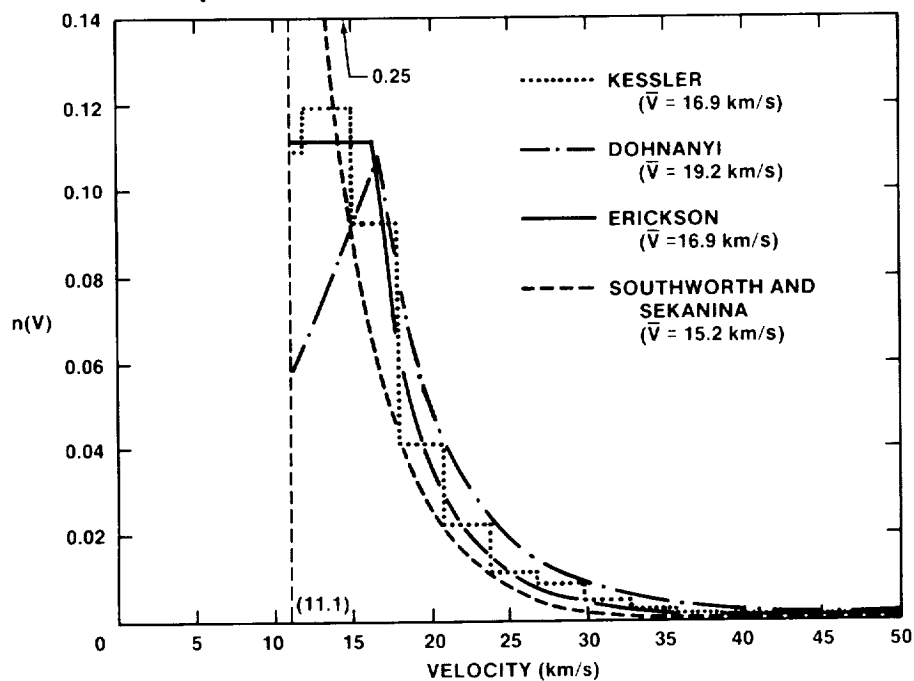


Figure 2. Velocity distributions of meteoroids entering the terrestrial atmosphere as independently corrected to constant meteoroid mass by different investigators (taken from Zook, 1975).

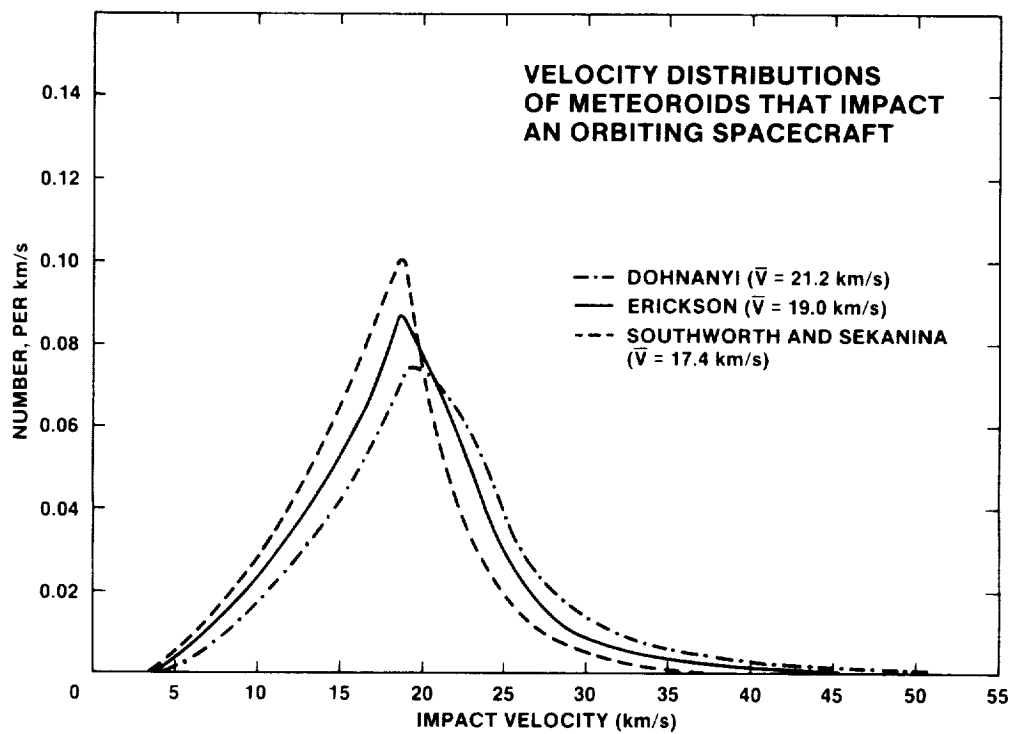


Figure 3. The velocity distributions of Figure 2 transformed to impact velocity distributions on spacecraft orbiting at 460 km above the Earth.

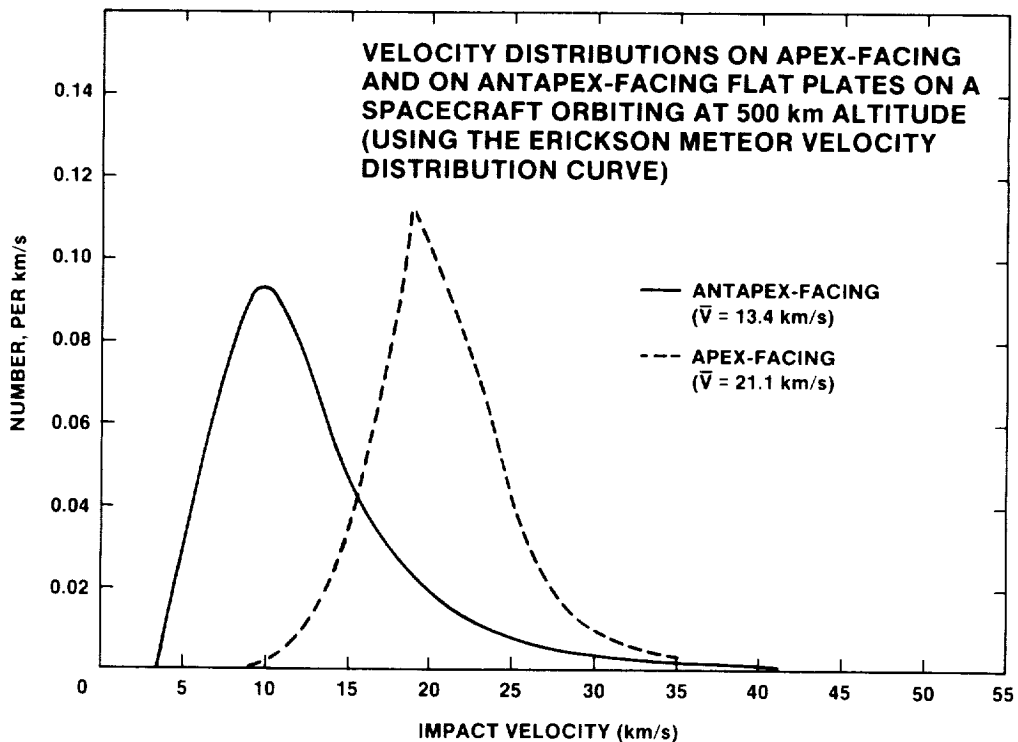


Figure 4. The Erickson-Kessler velocity distribution as transformed to velocity distributions on apex-facing and antapex-facing flat plates on a 460 km altitude orbiting spacecraft.

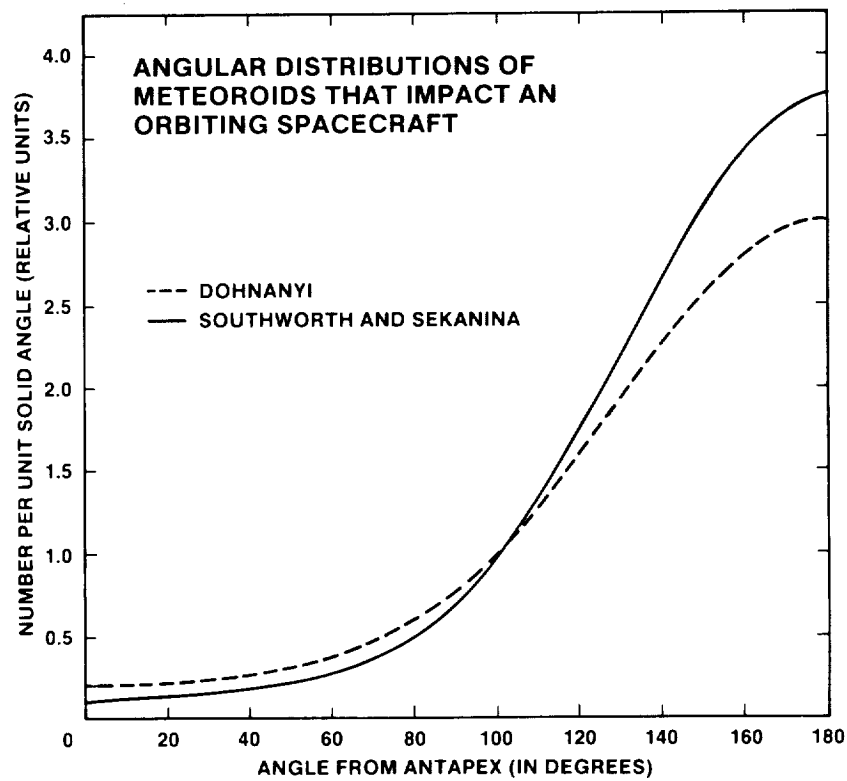


Figure 5. The flux of meteoroids per unit solid angle that approach a spacecraft orbiting at 460 km above the Earth versus angle from the apex direction. The fluxes are valid only for directions not shielded by the Earth and are those derived at constant meteoroid mass by Dohnanyi and by Southworth and Sekanina. The Erickson-Kessler distribution would lie in between the other two.

